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Prediction of the pulsation frequency of flames formed over a semi-infinite horizontal surface

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Abstract—The pulsation frequency of horizontally situated semi-infinite reacting surfaces is studied using linear stability theory. The analysis considers the flow to consist of an initially laminar base flow and a periodic disturbance. The governing equations are cast into a self-similar form analogous to previous analyses of boundary layers. The base flow is analyzed in terms of a flame sheet approximation, the solution to which is required to solve the disturbed flow. Some boundary conditions pertaining to the disturbances are evident but some others are inferred herein. The governing equations are solved for a flame burning *n*-heptane, and the results of the analysis relate the disturbance frequency of the flame to a Grashof type number which, in turn, is related to the length of the thermal boundary layer. It is determined that the frequency scales according to a power law expressed as $(\text{length})^{-0.8}$.

INTRODUCTION

IT HAS BEEN extensively observed that buoyant fires form large scale structures which are shed at a characteristic frequency (cf. [1–5]). Several attempts have been made to relate this pulsation frequency to the fire dimensions and characteristics, such as a characteristic diameter and heat release [1, 3, 5], Schonbucher *et al.* [3] conduct a holographic interferometric study of organized structures in a buoyant fire and determine one monoperiodic process associated with phenomena occurring close to the fire surface. Their study reveals a thermal boundary layer that includes within it axial and radial parcels of cold fuel or entrained air. It has been hypothesized that ring vortices, formed around the fire neck, pull up the outer edge of the fire to form an initial instability [5]. In the only theoretical work on the issue to date, Bejan [6] examines the problem basing his analysis on the buckling theory of inviscid streams [7]. Experimental studies on the flame spread of fires over horizontally stationed surfaces reveal that burning initially occurs in a 'boundary layer mode', and later in a 'plume mode' [8]. This implies the existence of length scales, smaller than a critical value, at which horizontally established buoyant flames will burn in a boundary layer mode. Therefore, the monoperiodicity that is to be found in fires formed over horizontally stationed surfaces may *partially* be due to instabilities in the thermal boundary layer, such a layer being similar to one surrounding the lower region of fires. The instabilities that are formed as a result may propagate downstream in a periodic fashion gaining in amplitude as heat is released. The present approach is similar to ones followed previously in order to analyze the flickering of an infinite candle [9], or that of wind-aided flame spread across a ceiling [10]. The method of analysis followed herein investigates how a given frequency interacts with the flow in order to determine a characteristic frequency corresponding to specific local flow conditions (cf. [11–13]).

In this study we address an idealized problem pertaining to the stability of a buoyant boundary layer existing in a steady flow. Such a boundary layer could be reasonably expected to exist along some space curve anchored at the outer edge of a buoyant fire. We recognize that such a layer would separate at some distance from the fire edge to become the outer sheath of a buoyant plume. Therefore, the study of the pulsation frequency of an actual fire must be coupled with an unsteady outer flow, which is outside the scope of a stability analysis such as the one presented herein. Having stated that, there is considerable merit in studying the idealized reacting flow problem that is contained herein. That merit lies in recognizing the role of fundamental boundary layer instabilities in reacting flows, the nonreacting counterparts of which have been extensively studied (cf. [7]). Though this problem is inapplicable to a fire in its entirety, it does, nevertheless, apply to disturbances along, and in the direction of the flame. The mathematical implication is that a parabolic flowfield is considered, rather than a set of elliptic governing equations.

The stability of thermal boundary layers has been extensively studied by Gebhart and co-workers, a review of which is to be found in Gebhart *et al.* [11]. These studies have successfully predicted the stability regions for horizontal and vertical natural convection flows in agreement with measurements. Our essential methodology follows their work, but, whereas they

NOMENCLATURE

- A quantity appearing in equation (8), $(g/5\nu_e^2)^{1/5}$
- $\bar{A}$ constant, from equation (20)
- a'_F stoichiometric coefficient associated with fuel
- a'_O stoichiometric coefficient associated with oxidizer
- a''_P stoichiometric coefficient associated with products
- B_0 frequency factor associated with the Arrhenius type expression of equation (7)
- c_p specific heat at constant pressure, assumed constant, dimensional
- D Damköhler number
- $\breve{D}$ modified Damköhler number
- d surface diameter
- F dimensional frequency
- f dimensionless stream function
- G modified Grashof type number, defined in equation (13)
- Gr_x Grashof type number of the form $Gr_x = (gx^3/\nu_e^2)$
- g gravitational acceleration, dimensional
- k thermal conductivity, assumed constant, dimensional
- M_F molecular weight of the fuel
- P instantaneous pressure, dimensional
- p pressure difference between the surface and the ambient, dimensional
- p' fluctuating pressure difference, dimensional
- Pr Prandtl number
- Q heat release due to stoichiometric combustion of the fuel, dimensional
- q latent heat of vaporization of the evaporating fuel, dimensional
- s disturbance temperature, dimensionless
- T dimensional temperature prior to equation (7), dimensionless following equation (7)
- t fluctuating temperature, dimensional
- T_a activation energy associated with the Arrhenius type expression of equation (7), dimensional prior to that equation, and dimensionless following it
- U^* characteristic velocity, dimensional
- U instantaneous x-wise velocity, dimensional
- u velocity component in the x-wise direction, dimensional
- u' fluctuating velocity component in the x-wise direction, dimensional
- v velocity component in the y-wise direction, dimensional
- v' fluctuating velocity component in the y-wise direction, dimensional
- $W_i h_i$ instantaneous contribution of the ith species to the overall reaction rate, dimensional
- x horizontal direction, perpendicular to the gravitational vector, dimensional
- Y_F mass fractions of fuel
- Y_O mass fractions of oxidizer
- y vertical direction, aligned with the gravitational vector, dimensional.

Greek symbols

- α complex wave number, dimensionless
- β volumetric coefficient of thermal expansion
- δ boundary layer thickness, dimensional
- Γ instantaneous quantity
- γ 'mean' quantity
- γ' 'fluctuating' quantity
- Θ instantaneous temperature, dimensional prior to equation (7), and dimensionless following it
- θ the ratio T/T_e
- ι $\sqrt{-1}$
- λ disturbance wavelength, dimensional
- μ dynamic viscosity, dimensional
- ν kinematic viscosity, dimensional
- $\Pi(\eta)$ dimensionless pressure
- $\boldsymbol{\rho}$ instantaneous density, dimensional
- ρ density, dimensional
- $(\Sigma\omega_i h_i)$ heat release due to the chemical reaction, dimensional
- $(\Sigma\omega_i h_i)'$ fluctuating heat release, dimensional
- Φ disturbance stream function, dimensionless
- Φ_1 integral, see equation (20)
- Φ_2 integral, see equation (20)
- Φ_3 integral, see equation (20)
- ψ dimensional stream function
- ω dimensionless frequency
- ω_i reaction rate associated with the production or destruction of the ith species, dimensional.

Subscripts

- e ambient conditions
- e conditions at the flame
- i signifying imaginary
- r signifying real
- w conditions at the horizontal surface.

did not consider the effect of chemical reaction, we do so by invoking the flame sheet approximation as described below. The purpose of this study is to identify the characteristic frequency associated with the instability of a laminar thermal boundary layer occurring over a horizontal evaporating surface involving chemical reaction, such as in an idealized buoyant fire. The horizontal surface is considered to be semi-infinite in extent, in accord with previously asserted assumptions involving buoyant flows [11].

THEORETICAL ANALYSIS

The flow configuration that is considered is schematically described in Fig. 1. An evaporating surface, hereafter referred to as the surface, is stationed horizontally, perpendicular to the gravitational vector. The surface temperature of the surface is at the fluid boiling point temperature T_w, the ambient temperature being denoted by the symbol T_e. Between the surface and the boundary layer edge lies a flame with a temperature T_f. The analytical methodology follows that of earlier studies [9, 11–13], to which the reader is referred for an extended description.

First, the boundary layer equations governing the flow are considered [11, 14], namely

$$\frac{\partial \rho u}{\partial x}+\frac{\partial \rho v}{\partial y}=0$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial p}{\partial x}$$

$$0=g\beta(T-T_e)-\frac{\partial p}{\partial y}$$

$$\rho u\frac{\partial T}{\partial x}+\rho v\frac{\partial T}{\partial y}=\frac{k}{c_p}\frac{\partial^2 T}{\partial y^2}-\sum_i w_i h_i. \qquad (1)$$

In equation (1) the Boussinesq approximation is invoked. We note that transformations of the Howarth–Dorodnitsyn type are available for the purpose of analysing flows with large density variations. In the present study such a transformation is used in order to obtain the 'base' solution pertaining to a flame sheet, as described below, following previous work [9, 14, 15]. However, the perturbed flow, also discussed below, is approximated as one in which the Boussinesq approximation is applicable. Such an assumption is appropriate for a first study; our later work will compare the results presented here with those obtained with a more complex analysis. In the third of the equations (1) viscous terms and all streamwise diffusion are neglected, an assumption in accord with previously conducted investigations of nonreacting flows [11].

The boundary conditions are of the form

$$u(x,0)=0,\quad \rho v(x,0)=(\rho v)_w,\quad u(x,\infty)=0,$$

$$p(x,\infty)=0,\quad (\rho c_p v)_e\frac{\mathrm{d}T}{\mathrm{d}y}(x,0)=q(\rho v)_w,$$

$$T(x,\infty)=T_e,\quad T(x,0)=T_w. \qquad (2)$$

In the above equations, the product $(\rho\mu)$ is assumed constant, the Prandtl number is set to unity, and the Boussinesq approximation is invoked.

The instability in the boundary layer is analyzed in terms of a mean and a disturbance so that a quantity Γ is represented by

$$\Gamma(x,y,\tau)=\gamma(x,y)+\gamma'(x,y,\tau) \qquad (3)$$

the prime denoting the disturbance. In the context of equation (3) the quantities u, v, ρ, p, and T that appear in equations (1) and (2) are represented by γ.

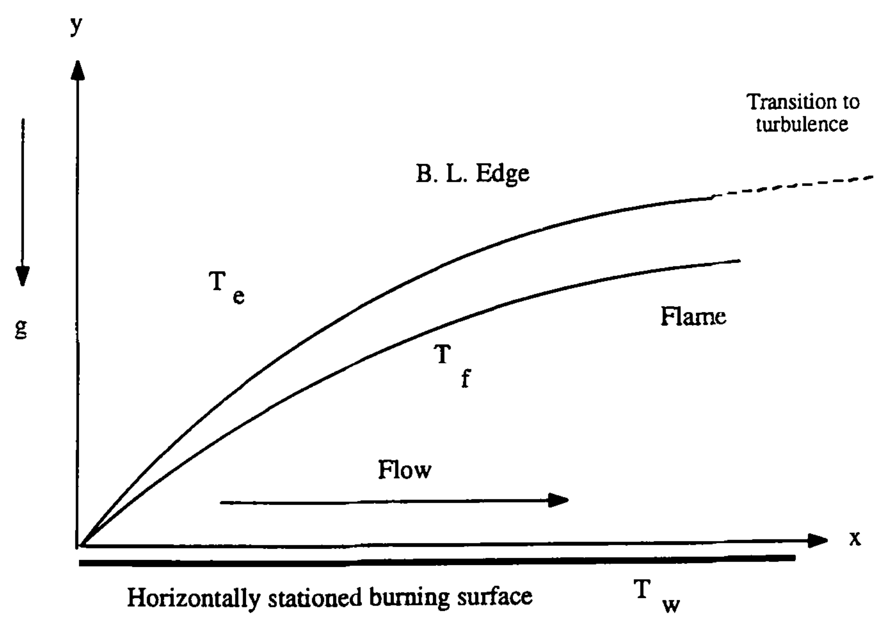

FIG. 1. Schematic diagram of a flame established over a semi-infinite horizontal surface.

Equations (1) and (2) may be thought of as being characteristic of a base flow. The full equations involving the corresponding quantities Γ, i.e. U, V, ρ, P, Θ and $W_i h_i$, are [11]

$$\frac{\partial \rho U}{\partial x}+\frac{\partial \rho V}{\partial y}=0$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=\nu\frac{\partial^2 U}{\partial y^2}-\frac{1}{\rho}\frac{\partial P}{\partial x}$$

$$\frac{\partial V}{\partial \tau}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}=\frac{g\beta}{\rho}(\Theta-\Theta_e)-\frac{1}{\rho}\frac{\partial P}{\partial y}$$

$$\frac{\partial \Theta}{\partial \tau}+U\frac{\partial \Theta}{\partial x}+V\frac{\partial \Theta}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 \Theta}{\partial y^2}-\frac{1}{\rho}\sum_i W_i h_i. \quad (4)$$

Next, the expressions obtained by substituting equation (3) into equation (4) are simplified by making the following approximations [11]: (1) the base flow is that represented by boundary layer theory (equations (1) and (2)); (2) terms of second-order, or higher, are discarded; (3) the parallel flow approximation is invoked, which implies that the mean component of v and the x-wise derivatives of the mean components of u and T are negligible; and (4) the amplification of the disturbance and its wavelength depend on the y-coordinate alone. As a result, the following equations are obtained:

$$\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}=0$$

$$\frac{\partial u'}{\partial \tau}+u\frac{\partial u'}{\partial x}+v'\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u'}{\partial y^2}-\frac{1}{\rho}\frac{\partial p'}{\partial x}$$

$$\frac{\partial v'}{\partial \tau}+u\frac{\partial v'}{\partial y}=\nu\frac{\partial^2 v'}{\partial y^2}+\frac{g\beta}{\rho}t-\frac{1}{\rho}\frac{\partial p'}{\partial y}$$

$$\frac{\partial t}{\partial \tau}+u\frac{\partial t}{\partial x}+v'\frac{\partial \theta}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 t}{\partial y^2}-\frac{1}{\rho}(\Sigma_i\omega_i h_i)' \quad (5)$$

where u', v', p', t and $(\Sigma\omega_i h_i)'$ represent the fluctuating quantities γ'. A one-step chemical reaction is taken to be representative of the oxidation of fuel into products, i.e.

$$a'_F \text{ fuel}+a'_O \text{ oxidizer} \rightarrow a''_P \text{ products}. \quad (6)$$

The chemical reaction rate is characterized by the relationship expressed below, namely

$$w_i=\frac{B_0\rho^2}{M_F}Y_F Y_O \exp(-T_a/T). \quad (7)$$

The solution for the mass fractions is to be determined by solving the associated species equation (cf. [14]).

Next, the temperature is made dimensionless by multiplying the energy equations in equations (1), (4) and (5) with the quantity c_p/Q. The original symbols for the temperature $\Theta = T+T'$ are retained for the dimensionless quantity itself; correspondingly, the activation energy is also made dimensionless and the symbol T_a retained. A self similar solution is sought for the governing equations, similarity being applicable since, as will be explained, the solution depends on the integration of the associated flame sheet problem. A dimensionless similarity variable may be defined as [11, 14]

$$\eta = Ax^{-2/5}\int_0^y \frac{\rho}{\rho_e}\,dy \quad (8)$$

along with a stream function that satisfies the continuity equation where the quantity $A = (g/5\nu_e^2)^{1/5}$. Making the assumption that

$$\psi = 5\nu_a A x^{3/5} f(\eta),$$
$$p(x,y) = 5\rho_e \nu_e^2 A^4 x^{2/5}\Pi(\eta), \quad T=T(\eta) \quad (9)$$

and proceeding in the usual manner (cf. [11, 14]), the following equations may be obtained that describe the base flow

$$f'''+3ff''-(f')^2-\frac{2}{5}\frac{T}{T_e}(\Pi-\Pi')=0$$

$$\Pi' = T/T_e-1$$

$$T''+3Pr\, fT' = -DY_O Y_F \exp(-T_a/T)$$

$$\eta\to\infty \quad f'=0, \quad T=T_e, \quad \Pi=0$$

$$\eta=0 \quad f'=0, \quad T=T_w, \quad T'_w=-qf_w \quad (10)$$

the primes denoting differentiation with respect to η. The Damköhler number is expressed by the following relation, i.e.

$$D=\frac{B_0\rho}{M_F\nu_e A^2 x^{-4/5}}. \quad (11)$$

Equations (10) are solved by considering the related flame sheet problem [14–17]; the solution of this specific problem is found in Puri [14] for three different fuels, i.e. n-heptane, toluene, and methanol. Obviously, in the flame sheet problem, the effects of finite rate kinetics are not considered, similar to an approach used before for a combustion boundary layer [9].

The equations characterizing the fluctuations, i.e. equations (5), are considered and the solution method closely follows earlier analyses [9, 11, 13]. The disturbance stream function and the temperature are described in terms of the dimensionless complex function Φ and s (that, as noted above, are assumed to be independent of the x-wise coordinate) by the following equations, i.e.

$$\psi'(x,y,\tau)=\delta U^*\Phi(\eta)\exp[i(\bar{\alpha}x-\bar{\omega}\tau)]$$
$$t(x,y,\tau)=s(\eta)\exp[i(\bar{\alpha}x-\bar{\omega}\tau)] \quad (12)$$

where

$$\delta(x)=\frac{5x}{G} \quad \frac{U^*x}{\nu_e}=\frac{G^2}{5} \quad G=5\left[\frac{Gr_x}{5}\right]^{1/5}$$

$$\alpha=\bar{\alpha}\delta=\bar{\alpha}_r\delta+i\bar{\alpha}_i\delta=\frac{2\pi\delta}{\lambda}+i\bar{\alpha}_i\delta=\alpha_r+i\alpha_i$$

$$\omega=\frac{\bar{\omega}\delta}{U^*}=\frac{2\pi F\delta}{U^*}=\omega_r. \quad (13)$$

Substitution of equations (12) and (13) into equations (5) yields the characteristic Orr–Sommerfield-type equations [11, 13] for the fluctuating quantities, namely

$$(f'-\omega/\alpha)(\Phi''-(\alpha\theta)^2\Phi)-f'''\Phi = \frac{\Phi''''-2(\alpha\theta)^2\Phi''+(\alpha\theta)^4\Phi}{\iota\alpha G\theta^2}-\frac{s}{5T}$$

$$(f'-\omega/\alpha)s-\Phi T' = \frac{s''-(\alpha\theta)^2 s}{\iota\alpha Pr\, G\theta^2}-\ddot{D}Y_O Y_F \exp(-T_a/T)\frac{T_a s}{T^2}. \quad (14)$$

In equation (14) the primes denote differentiation with respect to η, and $\ddot{D}$ is a modified Damköhler number that can be expressed as

$$\ddot{D} = \frac{B_0\rho Pr}{M_F v_c (G/4x)^2}. \quad (15)$$

If the fluctuations involving the temperature are small such that $(s/T) \ll (T/T_a)$, then, in any event, the last term in the second of equations (14) may be neglected. Near the fuel surface in buoyant fires, the region to which this analysis is expected to apply, the experimental results of Fischer [18] indicate that this is indeed the case so that we neglect the contribution due to disturbances in the reaction rate. Again, since the present analysis is an idealization, this comparison is introduced in a 'rough' sense. The ambient is undisturbed, so that the boundary conditions related to equations (14) in the ambient are

$$\Phi(\infty) = \Phi'(\infty) = s(\infty) = 0,$$

such that

$$\Phi_r(\infty) = \Phi_i(\infty) = \Phi_r'(\infty) = \Phi_i'(\infty) = s_r(\infty) = s_i(\infty) = 0. \quad (16)$$

The other boundary conditions are not immediately obvious and must be inferred intuitively. We assume that disturbances at the surface are minimal, which is reasonable in light of the large thermal capacity of the surface. Therefore

$$s(0) \approx 0, \quad \text{or} \quad s_r(0) \approx s_i(0) \approx 0. \quad (17)$$

Other experimental measurements on buoyant natural gas fires [19] imply that the quantities (r.m.s. u')/$u \approx 0.6$ and (r.m.s. v')/$v \approx 0.7$ very near the surface, the terms (r.m.s. u') and (r.m.s. v') having values equal to $(u'^2)^{1/2}$ and $(v'^2)^{1/2}$, respectively. In light of these observations we assume that for our idealized case

$$|\Phi_r(0)| \approx 0.7 \quad |\Phi_r'(0)| \approx 0.6. \quad (18)$$

In the context of equations (18) we note that pressure waves have been observed on the surface of gaseous buoyant fires [20]. The boundary conditions represented in equation (18) are readily substituted, without loss of generality in the analysis; only the resulting solutions will be different. We note that the terms f', f''', T and T' appearing in equations (14) are obtained from the solution of equations (10) following the earlier analyses referenced above. We note that applying the observed boundary conditions does not analyze the inherent stability of the flame, but determines conditions corresponding to the observed instability (cf. [11–13]).

Heiber and Gebhart [21] elucidate a method that has been extensively used for the solution of equations (14) considering nonreacting flows [11–13]. For reacting flows the system of equations described by equations (10) and (14) is highly nonlinear, so their method is applied after some modification. The above mentioned system of equations possesses three linearly independent integrals which are negligible at infinity [21]. Thus, as $\eta \to \infty$, a choice of these integrals is made such that

$$\Phi_1 = \exp[-(\alpha\theta)\eta]$$
$$\Phi_2 = \exp[\surd(-(\alpha\theta)^2-\iota\omega G\theta^2\eta)]$$
$$\Phi_3 = \exp[\surd(-(\alpha\theta)^2-\iota\omega G Pr\theta^2\eta)]. \quad (19)$$

In this analysis we assume that the Prandtl number equals unity and note that as $\eta \to \infty$ the quantity θ approaches a value equal to one. Upon application of these approximations the composite solution may be written in the form

$$\Phi = \Phi_1 + \bar{A}\Phi_2 \quad (20)$$

which must satisfy the boundary conditions at the surface, i.e. the first of equations (18), at $\eta = 0$ so that the constant $\bar{A}$ is easily obtained. Further, equation (20) may be simplified in terms of its real and imaginary parts and expressed as

$$\Phi = \exp(-\alpha\eta)(1+\bar{A}\cos(\kappa\eta))+\iota\bar{A}\exp(-\alpha\eta)\sin(\kappa\eta),$$
$$\kappa = \frac{G}{2}(\omega/\alpha). \quad (21)$$

The second derivative of equation (21), with the constraint that $\Phi''' = 0$ (since $f''' = 0$ for the base solution), is employed to start the solution at $\eta \to \infty$ in order to shoot for the boundary conditions expressed in equations (17) and (18). This imposes two additional conditions on equations (14) from which the disturbance frequency ω and amplitude α are ascertained.

RESULTS AND DISCUSSION

While the boundary conditions imposed through the application of equations (17) and (18) are specific to values obtained from the literature, we stress that this analysis is generic and independent of experimental measurements. For instance, in light of uncertainty in the exact boundary conditions that are used, one can change the nature of equations (17) and (18) and still apply the above analysis both in form and content; the results, though, are expected to be effected by these changes. For the sake of simplicity

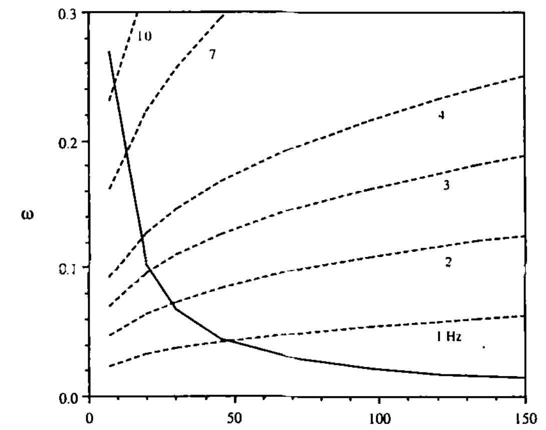

FIG. 2. The stability limits describing the variation of the dimensionless frequency with G (solid curve), with lines of constant frequency (dashed curves) represented in the domain.

we consider semi-infinite horizontal surfaces burning a fuel that has the same thermodynamic, chemical and transport properties as n-heptane in an ambient consisting of atmospheric air. The base solution for such fires has been previously determined [14] and is the same as that used in this study. The salient physical and chemical properties of n-heptane are briefly recalled: the latent heat of vaporization L is 7.50 kcal mol^{-1}; the heat release Q is taken to be 1068 kcal mol^{-1}; the ratio of specific heat to heat release c_p/Q is taken as 4.7×10^{-5}; the peak temperature is 2396 K; the ambient kinematic viscosity is taken to be 0.157 cm^2 s^{-1}; and the temperature of the surface, taken as the boiling point temperature of the liquid, is 371 K.

The disturbance amplitude α is held fixed and that of ω varied in order to match the boundary conditions at $\eta = 0$. As this procedure is repeated for several values of G, an outer stability region in terms of ω and G is described which signifies a unique characteristic dimensionless frequency for a particular value of G. The term G is in turn related to the boundary layer length scale (cf. equations (13)). The value of α was held fixed for this study, and kept equal to 0.5, which implies, for example, an amplitude of roughly 5 cm at a distance of 10 cm inwards from the surface edge. A similar inference has been previously made for the stability of an infinite candle [9]. Previous studies pertaining to nonreacting flows [11, 13] demonstrate this value to be plausible since it lies within the stable region. In any event, the calculated results are almost unchanged if the value ascribed to α is doubled.

The numerical solution relating ω to G is reported in Fig. 2 in which the solid curve represents the lower stability limits. The interpretation of the results is that for a particular value of G there is a distinct value of ω at which the buoyant fire first becomes unstable. The upper stability limits [11–13] were not calculated since it is the monoperiodicity of buoyant fires that is our concern. With a little algebraic manipulation, using the definitions outlined in equations (13), it can be shown that

$$\omega G^{-1/3} = 2\pi F/[\nu_c(5g/\nu_c^2)^{2/3}] \quad (22)$$

which enables lines of constant frequency to be calculated; these lines are represented by dashed curves in Fig. 2. Following a constant frequency line, a characteristic value of G is determined when the stability limit curve is intersected.

If the thermal boundary layer is assumed to extend to an x-wise length L which is less than the surface radius, the terms G and Gr_x may be calculated (cf. equations (13)). In Fig. 3 we interpret the results presented in Fig. 2 by relating the dimensionless disturbance frequency to the boundary layer length where, again, lines of constant frequency are also plotted. Thus, a characteristic monoperiodic frequency may be determined for any boundary layer length that is considered. If a fit is made to describe ω in terms of L, the resultant expression obtained is of the form

$$\omega \cong 0.0675L^{-0.6}. \quad (23)$$

The length L is implicitly present in the definition of ω (cf. equations (13) and (22)), and with the appropriate substitutions equation (23) may be written in a form that relates the dimensional frequency F to the boundary layer length, i.e.

$$F \cong 0.2045(g^{0.6}/\upsilon_c^{-0.2})L^{-0.8}. \quad (24)$$

An appropriate value for L, that may be considered in equation (24), is $L = C(d/2)$ where C is a constant and d the surface diameter. Substitution of this value into equation (24), provides a relation in terms of the surface diameter. In the absence of rigorous data the constant C must be empirically determined from results obtained from experimental measurements.

In equation (24) the value of L is expressed in units

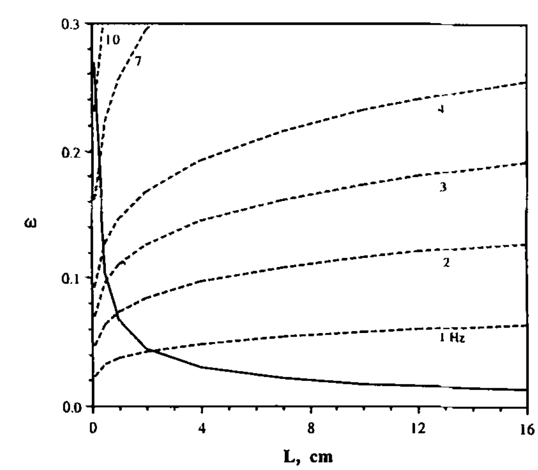

FIG. 3. The stability limits describing the variation of the dimensionless frequency with the x-wise thermal boundary layer length, corresponding to Fig. 2, calculated from equation (13) (solid curve) with lines of constant frequency (dashed curves) represented in the domain.

of cm. Though the above relation has been derived for semi-infinite surfaces burning liquid *n*-heptane as fuel, it has been experimentally observed that the disturbance frequency correlates well with surface diameter regardless of the nature of the fuel [5], though, as stated earlier, buoyant fires are described more fully by a set of elliptic equations. The predictions also indicate that the pulsation frequency depends on the gravitational constant as ($g^{0.6}$), thereby implying that reduced gravity flames exhibit a much smaller pulsation frequency.

In closing, it is appropriate to reiterate the objectives of this work, so that any limitations are viewed in perspective. It is not contended herein that the *complete* mechanism of buoyant fire fluctuation is fully addressed by the present analysis. Rather, it is proposed that at length scales that are characteristic of this idealized problem, it is appropriate to consider burning over horizontal surfaces to occur in a thermal boundary layer.

CONCLUSIONS

An analytical study of the pulsation frequency of a flame established above a semi-infinite horizontal surface is conducted in terms of a base flow and a periodically occurring disturbance in a thermal boundary layer. A similarity solution obtained for the base flow forms the basis of analyzing the disturbance. The same similarity variable is used to derive the relevant equations governing the fluctuating flow. Some of the boundary conditions applicable to the disturbed flow are inferred from previously conducted experimental measurements pertaining to buoyant fires. A parabolic solution is considered appropriate due to the physics of the instability which is observed to grow in amplitude downstream from the surface but with an unchanging characteristic frequency. While a solution to the entire flowfield is not obtained, a perturbation analysis of the thermal boundary layer shows that the buoyant flame pulsation is related to a characteristic length scale, taken to be the surface diameter.

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